# Vers une approche non-paramétrique en mécanique des matériaux

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Math-erials, Grenoble, 01/2019.

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Find $(\mathbf{u}, \boldsymbol{\sigma})$ such that:

- Compatibility:

$$\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla \mathbf{u} + \nabla^T \mathbf{u})$$

- Constitutive relation:

$$\boldsymbol{\sigma} = f(\boldsymbol{\varepsilon}, \ldots)$$

- Balance of momentum

$$\mathrm{div}(\boldsymbol{\sigma}) = 0$$

- Boundary conditions

$$\mathbf{u} = \mathbf{u}_d \text{ on } \Gamma_u$$

$$\boldsymbol{\sigma}(\mathbf{n}) = \mathbf{F}_d \text{ on } \Gamma_F$$

# Continuum mechanics

Find $(\mathbf{u}, \boldsymbol{\sigma})$ such that:

- Admissibility for $\mathbf{u}$:

$$\mathbf{u} = \mathbf{u}_d \text{ on } \Gamma_u \text{ + regularity}$$

- Admissibility for $\boldsymbol{\sigma}$:

$$\mathrm{div}(\boldsymbol{\sigma}) = 0$$

$$\boldsymbol{\sigma}(\mathbf{n}) = \mathbf{F}_d \text{ on } \Gamma_F$$

- Constitutive relation:

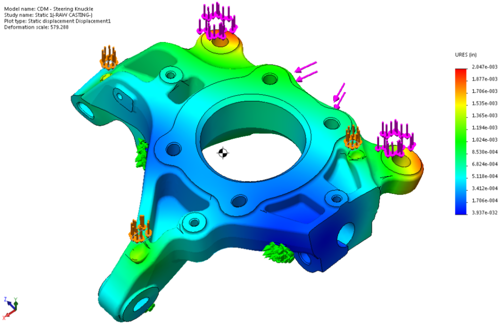
$$\boldsymbol{\sigma} = f(\boldsymbol{\varepsilon}, \ldots)$$

with $\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla \mathbf{u} + \nabla^T \mathbf{u})$

Resolution:

Minimization of an energy functional / Principle of Virtual Work

under constraint: $\boldsymbol{\sigma} = f(\boldsymbol{\varepsilon}, \ldots)$



# Limitations of constitutive laws

Complex Materials:

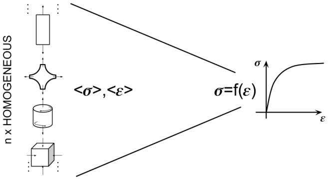
- continuum inherits all the complexity of the smaller scales

Advanced theoretical framework:

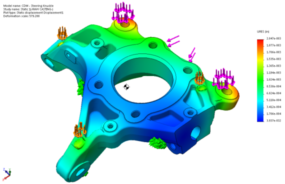
- non-linear, history dependent,...
- $\boldsymbol{\varepsilon}, \boldsymbol{\sigma}$ are $2^{nd}$ order tensors

Too simple experiments:

- homogeneous $\boldsymbol{\varepsilon}, \boldsymbol{\sigma}$
- mainly uniaxial




$\neq$



$\longrightarrow$ extrapolation

Find $(\mathbf{u}, \boldsymbol{\sigma})$ such that:

- Admissibility for $\mathbf{u}$:

$$\mathbf{u} = \mathbf{u}_d \text{ on } \Gamma_u \text{ + regularity}$$

- Admissibility for $\boldsymbol{\sigma}$:

$$\mathrm{div}(\boldsymbol{\sigma}) = 0$$

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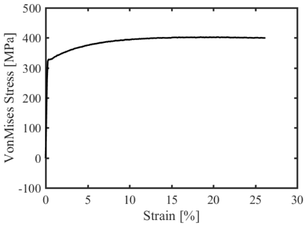
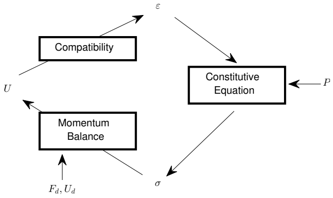
- Constitutive relation:

$$\boldsymbol{\sigma} = f(\boldsymbol{\varepsilon}, \ldots)$$

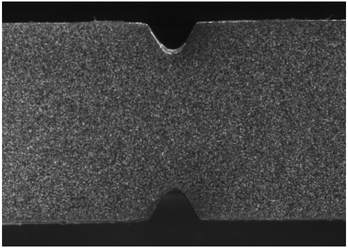
with $\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla \mathbf{u} + \nabla^T \mathbf{u})$

- "Engineering" approach



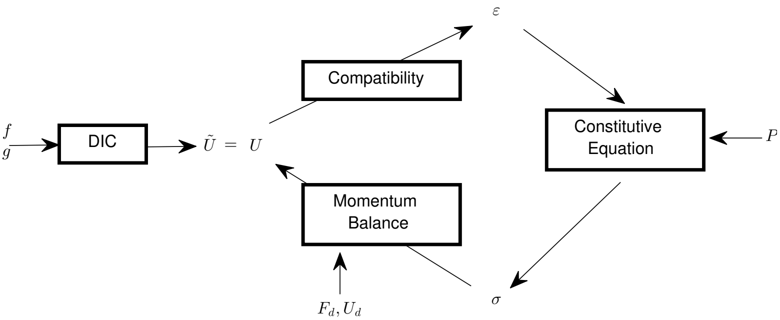



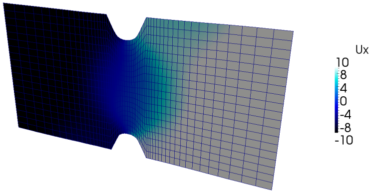

- Validation ?



by G. Portemont at ONERA Lille

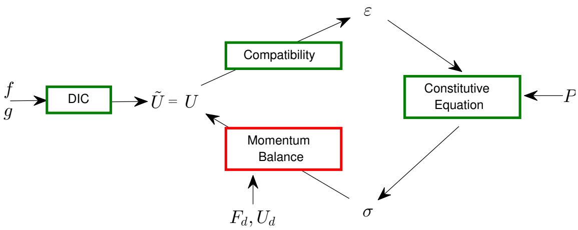
- Photomechanics



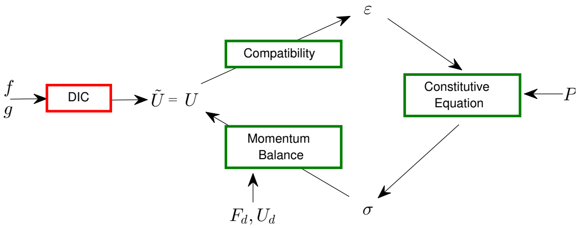



# Limitations of constitutive laws

- Stress calculation

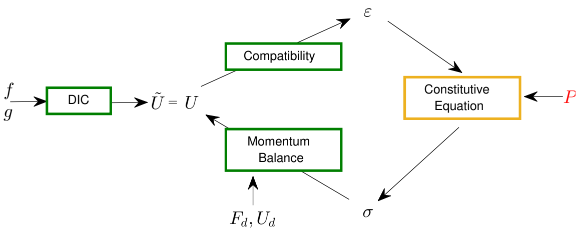



- Numerical simulation

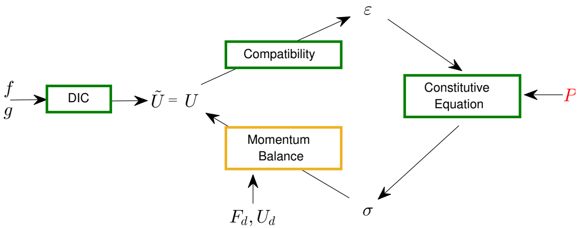


# Limitations of constitutive laws

- Constitutive Equation Gap [Chrysochoos *et al.*]

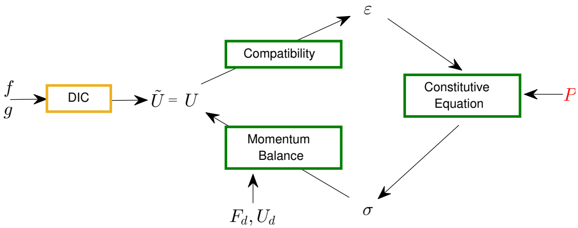


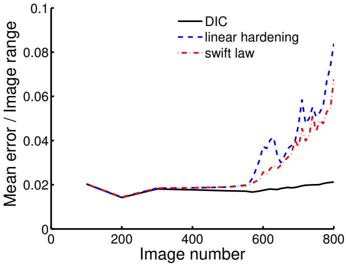

- Equilibrium gap [Hild *et al.*], VFM [Grédiac *et al.*]

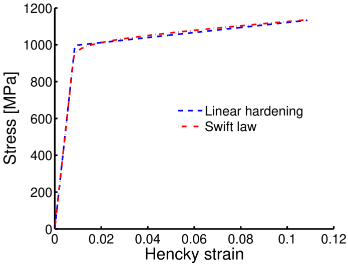


# LIMITATIONS OF CONSTITUTIVE LAWS

- FEMU [Lecompte *et al.*, Leclerc *et al.*...]

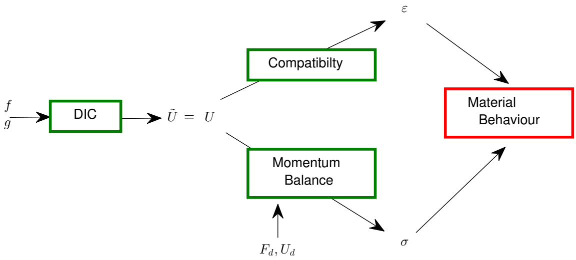


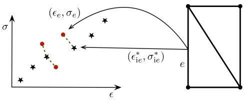



# Parametric v.s. Non-parametric

- Parametric techniques (using a constitutive equation)
  - \+ provide for the optimal set of parameters
  - +/- tell that the constitutive equation is not *correct*
  - \- how to improve it
  - \- kinematically consistent direct FEA
- Non-parametric (without using a constitutive equation)



# Towards a new paradigm using data beyond laws



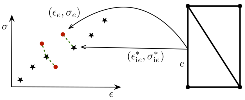

- use *data* beyond *laws*
- Min distance *w.r.* data under BE constraint
- Kirchdoerfer & Ortiz (2016)

data are obtained from *simple* experiments $\longrightarrow$ limitations remain

**Goals**

- estimate heterogeneous material state fields
  - strain
  - stress
  - …
- no pre-supposed constitutive equations

# Towards a new paradigm using data beyond laws

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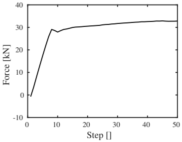
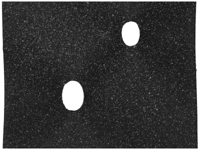
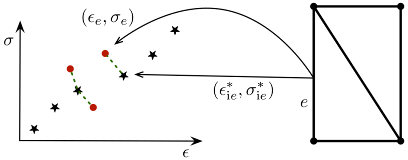
1. Inverse data-driven →non-linear elasticity

2. Mixed reduction →plasticity






- $(\epsilon^*_{\text{ie}}, \sigma^*_{\text{ie}})$ = material state
- $(\epsilon_e, \sigma_e)$ = mechanical state
- $ie$ mapping

Minimization of the *distance* between $(\epsilon_e, \sigma_e)$ and $(\epsilon^*_{\text{ie}}, \sigma^*_{\text{ie}})$

$$\text{solution} = \arg \min_{\epsilon_e, \sigma_e, \text{ie}} \frac{1}{2} \sum_e w_e \left( C_e(\epsilon_e - \epsilon^*_{\text{ie}})^2 + \frac{1}{C_e}(\sigma_e - \sigma^*_{\text{ie}})^2 \right)$$

subjected to:

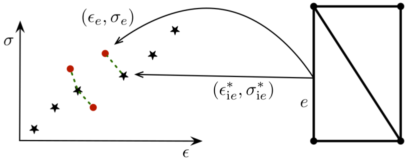
- Balance of momentum

$$\sum_e w_e \mathsf{B}_{ej} \sigma_e = \mathsf{f}_j$$

- Strain compatibility

$$\epsilon_e = \sum_j \mathsf{B}_{ej} \mathsf{u}_j$$

*written for truss lattice*

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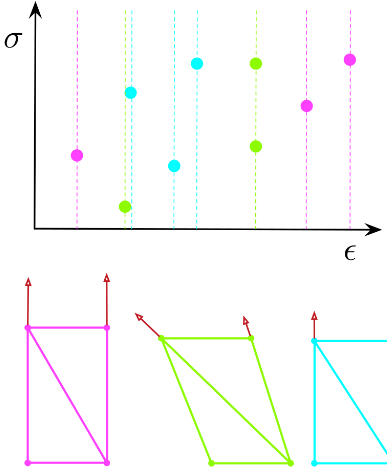
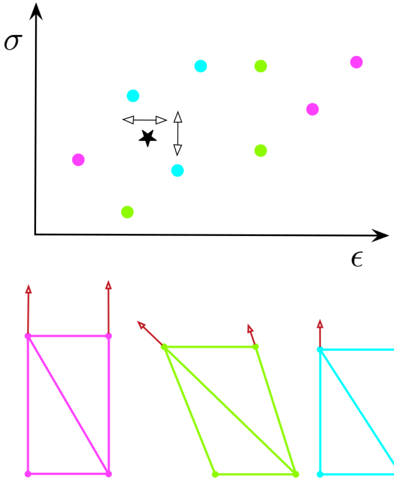
Starting from strain measurements




the mechanical points can only move vertically

# How it works....

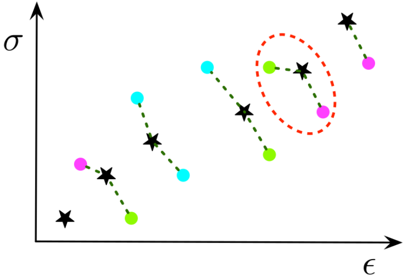
Starting from strain measurements



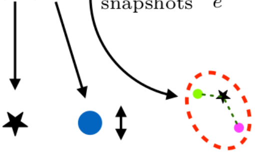

but the material states are unknown

# How it works....

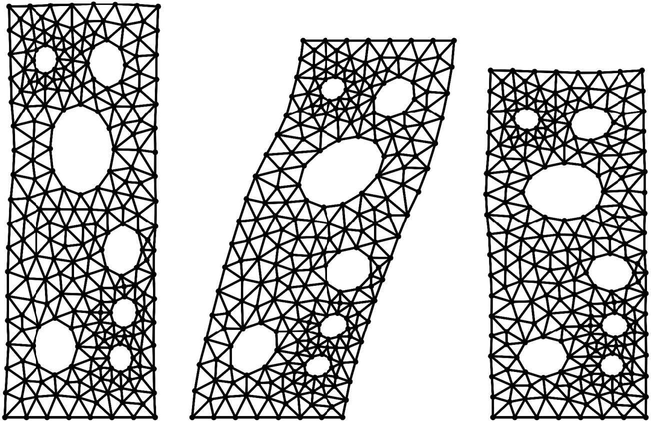
Starting from strain measurements




$$\min_{\epsilon_i^*,\sigma_i^*,\sigma_e,\mathrm{ie}} \sum_{\text{snapshots}} \sum_{e} \left( C_e(\epsilon_e - \epsilon_{\mathrm{ie}}^*)^2 + \frac{1}{C_e}(\sigma_e - \sigma_{\mathrm{ie}}^*)^2 \right)$$



Example (3 snapshots, 40 material states, 650 mechanical states)



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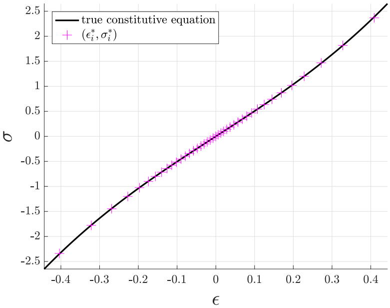
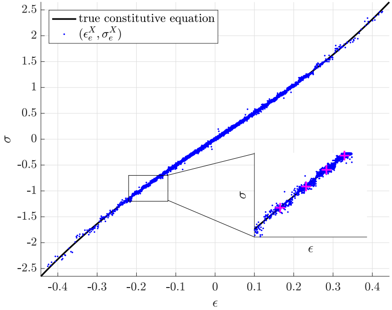
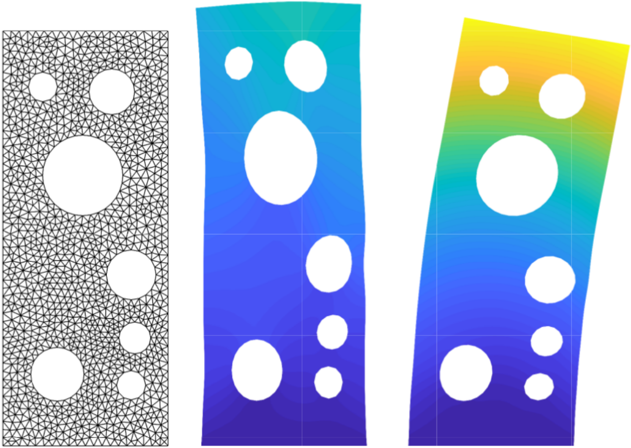


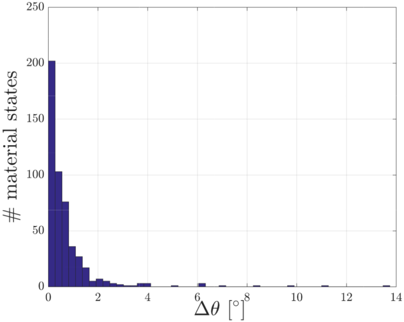
# 2D Plane stress hyper-elasticity (synthetic data)

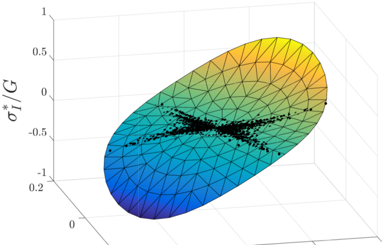
Setup (40 snapshots, 2400 elements)

$$\begin{aligned} \boldsymbol{\sigma} &= G(\boldsymbol{\epsilon} + \alpha \boldsymbol{\epsilon}^3) - p\mathbf{I} \ , \\ p &= -(\epsilon_{xx} + \epsilon_{yy}) - \alpha(\epsilon_{xx} + \epsilon_{yy})^3 \ . \end{aligned}$$



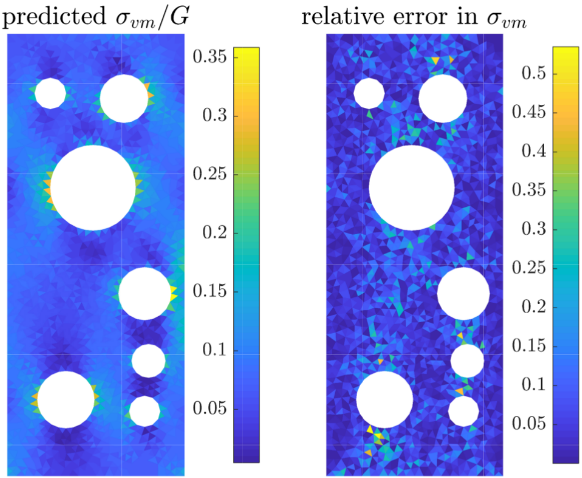
Material state database (500 states)



# 2D Plane stress hyper-elasticity (synthetic data)

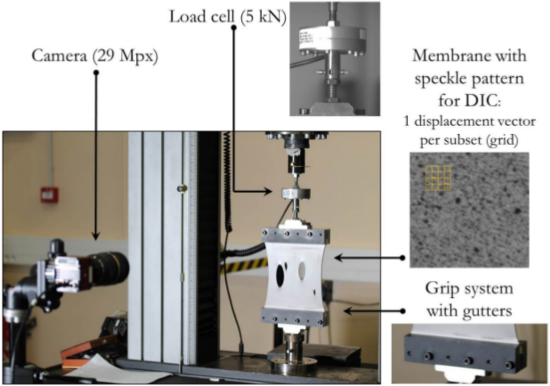
Stress reconstruction




A. Leygue *et al.*. Data-based derivation of material response. CMAME, 2018.

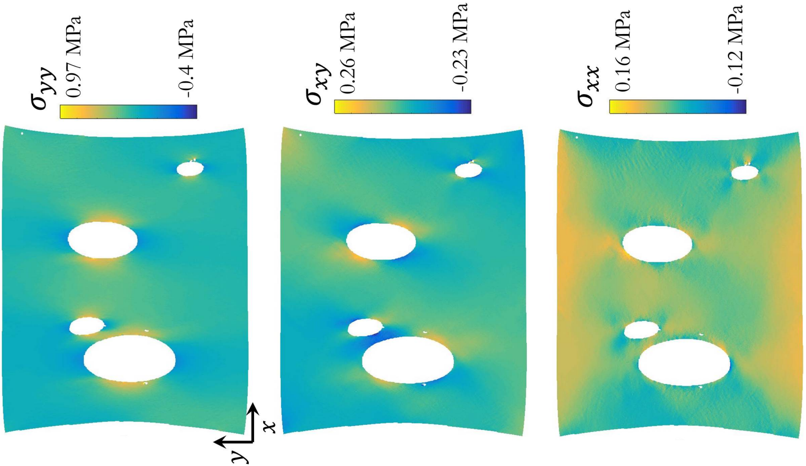
# Silicone rubber experiment (Marie Dalemat)

## Experimental setup



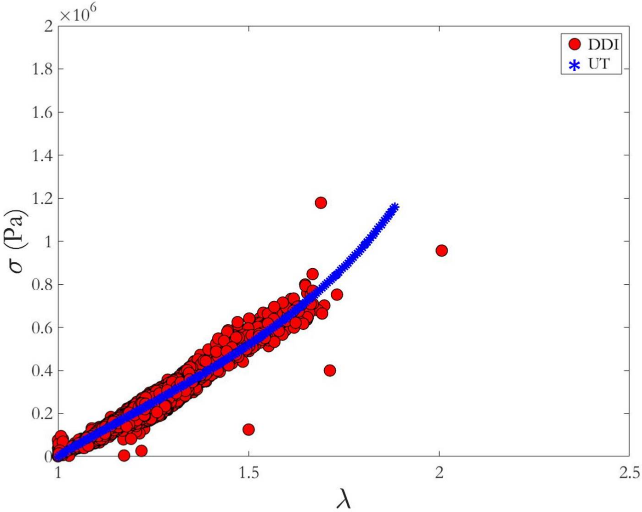

- Silicone rubber RTV ≈ non-linear elasticity + incompressibility
- 2D thin membrane ≈ plane stress
- 150 29 MPix images

Stress reconstruction (30 000 000 mechanical states)

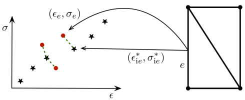


# Silicone rubber experiment (Marie Dalemat)

Comparison with uniaxial tensile test data (10 000 material states)




First principal component

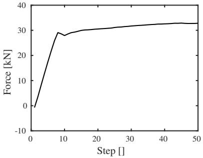


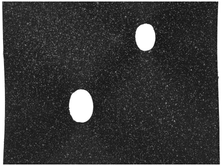

- use *data* beyond *laws*
- Min distance *w.r.* data under BE constraint
- beyond Kirchdoerfer & Ortiz (2016)

data are obtained from *simple* experiments $\longrightarrow$ limitations remain

1. Inverse data-driven $\rightarrow$non-linear elasticity

2. Mixed reduction $\rightarrow$plasticity



Assume a constitutive law has already been calibrated. One can compute the stress field $\tilde{\boldsymbol{\sigma}}_e^X$ balancing the external load. It differs from the actual one

$$\boldsymbol{\sigma}_e^X = \tilde{\boldsymbol{\sigma}}_e^X + \bar{\boldsymbol{\sigma}}_e^X$$

Only the correction $\bar{\boldsymbol{\sigma}}_e^X$ is searched for.

- $\bar{\boldsymbol{\sigma}}_e^X$ is self-balanced
- $\boldsymbol{\sigma}_e^X = \tilde{\boldsymbol{\sigma}}_e^X + \mathsf{L}_e^j \mathsf{P}_j^X$

The minimization problem is reformulated:

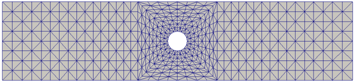
$$\text{solution} = \arg \min_{\mathsf{P}_j^X, \boldsymbol{\epsilon}_i^*, \boldsymbol{\sigma}_i^*, \mathrm{i}e^X} \mathcal{E}(\tilde{\boldsymbol{\sigma}}_e^X + \mathsf{L}_e^j \mathsf{P}_j^X, \boldsymbol{\epsilon}_i^*, \boldsymbol{\sigma}_i^*, \mathrm{i}e^X)$$

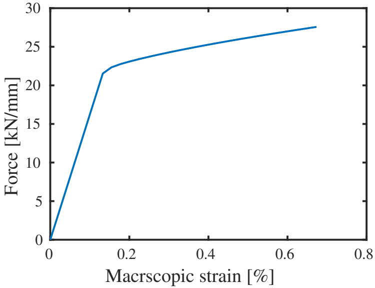
- No additional constraint
- The basis migh be reduced
- The correction might be local

Specimen and model for synthetic image generation:

$\mathcal{C}_o$:E=168 GPa, $\nu$=0.25

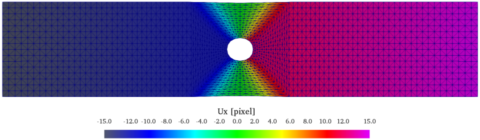
linear isotropic hardening: $\sigma_y$=285 MPa, H=1.48 GPa



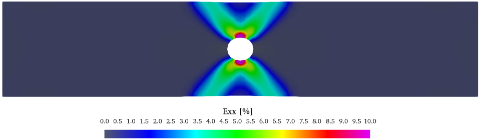



35 steps

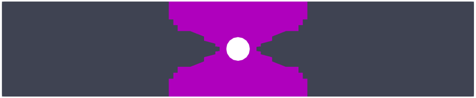
- Displacement




- Total strain

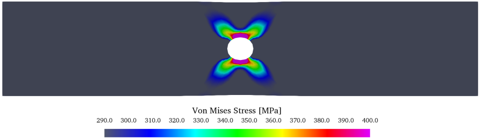



- Reduced zone (elements such that $||\varepsilon|| > 0.2\%$)

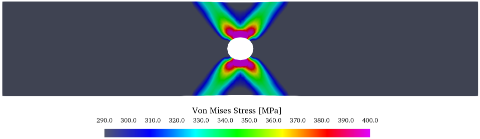


# Plasticity

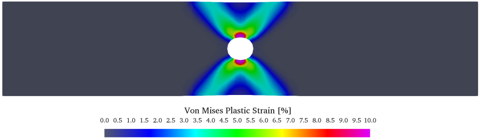
- Initial Von Mises stress (elastic)




- Corrected Von Mises stress



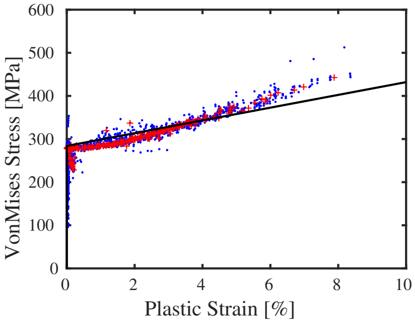

- Reconstructed Von Mises plastic strain using $\boldsymbol{\sigma}_{zz} = 0$ and $Tr(\boldsymbol{\varepsilon}_p) = 0$



Stress/Plastic strain plot:

full $\boldsymbol{\sigma}, \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}_p$ history

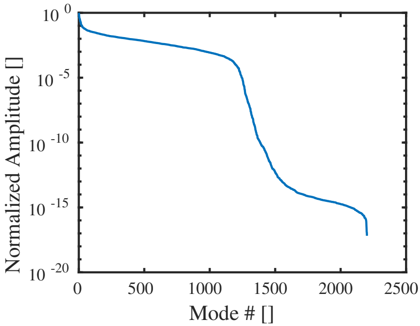
• mechanical states, + material states




$>$ 100000 points on a manifold of dim 6 in a space of dim 12 $(\boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \boldsymbol{\varepsilon}_p)$

Model reduction

$$\bar{\boldsymbol{\sigma}}_e^X = \mathsf{L}_e^j \mathsf{P}_j^X$$




*A priori* $3N_e^{RD} - 2N_n^{RD}$ modes $\longrightarrow$ only one half should be considered

# Conclusion

Highlights

- Stress field can be reconstructed from
  - DIC displacement fields
  - force measurements
- no parametric phenomenological relation is required

Possible improvement & prospects

- best database $\approx$ DDI for history dependent materials
- coupling with IRT (releasing the max diss. condition)
- non-proportionnal loadings
- ...

# Vers une approche non-paramétrique en mécanique des matériaux

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Math-erials, Grenoble, 01/2019.

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