Séminaire Math-ériaux

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Sur l'utilisation des invariants en optimisation des Matériaux et des Structures

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Abstract

L'exposé traitera de l'utilisation des invariants dans une démarche simultanée d'optimisation topologique et de distribution de l'anisotropie de structures élastiques. En 2D, la description de l'anisotropie sera basée sur l'utilisation conjointe d'invariants polynomiaux (base d'intégrité) et d'une paramétrisation par nombres complexes (formalisme polaire). Une méthodologie d'optimisation hiérarchique utilisant le lien entre invariants et micro-structure sous-jacente sera décrite et appliquée au cas des structures minces composites stratifiées 2D. Enfin, une extension de la méthode aux structures élastiques 3D constituées de matériaux isotropes transverses sera détaillée.

Physical motivations

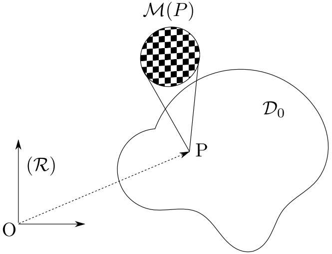
The elasticity tensor is an image of the microstructure of the material.

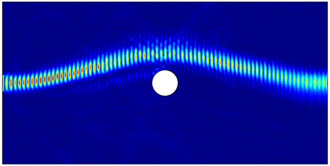
Optimal design

- Response to specifications;
- Achieving non standard properties;
- Controlling wave propagation;
- ...

Evolution of the matter

- Damaging of the matter;
- Growth of bio-material;
- Control of smart materials;
- ...





Optimal design using invariants

2D anisotropic Elasticity, small strains, small displacements

Structural problem

$\updownarrow$

Equivalent behavior
Invariant parametrization

$\updownarrow$

Micro structure level
Geometrical parametrization

Structural optimization

Objective function : Mass, Buckling, Energy
Optimization parameters : Invariants

Optimal design of achitectured materials

Objective function : f(Invariants)
Optimization parameters : Geometry

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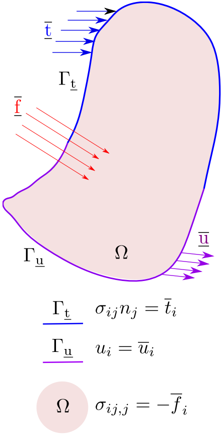
What is elasticity ?

Consider :

- a domain $\Omega \subset \mathbb{R}^d$;
- $\underline{u}$ the displacement field over Ω;
- $\underline{f}$ the bulk force field over Ω;
- ε the strain field over Ω;
- σ the stress field over Ω;

Ω is at rest provided:

$$\begin{cases} \underline{\text{div}}\sigma + \underline{f} = 0 \\ \sigma \cdot \underline{n} = \underline{\bar{t}} \end{cases}$$



Constitutive law

A constitutive model is needed to relate σ to ε

The elasticity tensor

Hooke's law

Linear relation between the stress tensor $\sigma \in S^2(\mathbb{R}^2)$ and the strain tensor $\varepsilon \in S^2(\mathbb{R}^2)$:

$$\sigma = \mathbb{C} : \varepsilon$$

Properties

$\mathbb{C}$ is an element of the vector space $\mathbb{E}\mathrm{la} := S^2(S^2(\mathbb{R}^2))$;
$\mathbb{C}$ is positive definite :

$$\forall \varepsilon \neq 0, \quad \varepsilon : \mathbb{C} : \varepsilon > 0$$

An elastic material: a O(2) -orbit

O(2)-action

O(2) acts on $\mathbb{E}\mathrm{la}$ through standard $\star$ defined by :

$$\begin{aligned} \star : \mathrm{O}(2) \times \mathbb{E}\mathrm{la} &\to \mathbb{E}\mathrm{la} \\ (\mathrm{Q}, \mathbb{C}) &\mapsto \mathrm{Q} \star \mathbb{C} := Q_{ip} Q_{jq} Q_{kr} Q_{ls} C_{pqrs} \end{aligned}$$

Orbit

The set of tensors of $\mathbb{E}\mathrm{la}$ O(2)-conjugate to $\mathbb{C}$ constitutes its O(2)-orbit :

$$\mathrm{Orb}(\mathbb{C}) := \left\{ \overline{\mathbb{C}} = \mathrm{Q} \star \mathbb{C} \mid \mathrm{Q} \in \mathrm{O}(2) \right\}.$$

The orbits space is the quotient space $\mathbb{E}\mathrm{la}/\mathrm{O}(2)$.

Symmetry Class

Symmetry Group

Symmetry group of an elasticity tensor:

$$\mathrm{G}_{\mathbb{C}} := \{\mathrm{Q} \in \mathrm{O}(2), \quad \mathrm{Q} \star \mathbb{C} = \mathbb{C}\}.$$

Symmetry Class

The class of symmetry is the conjugacy class of a symmetry group:

$$[\mathrm{G}_{\mathbb{C}}] := \left\{\mathrm{Q}\mathrm{G}_{\mathbb{C}}\mathrm{Q}^{-1}, \quad \mathrm{Q} \in \mathrm{O}(2)\right\}.$$

$\mathbb{E}$la is divided into strata of different symmetry classes.

2D Symmetry classes

In 2D, the space of elasticity tensors is divided into 4 strata:

$$\mathbb{E}\text{la} = \Sigma_{[\mathrm{Z}_2]} \cup \Sigma_{[\mathrm{D}_2]} \cup \Sigma_{[\mathrm{D}_4]} \cup \Sigma_{[\mathrm{O}(2)]}$$

- Z_2: cyclic group generated by $\mathrm{R}(\pi)$, a rotation of angle π;
- D_k: dihedral group generated by $\mathrm{R}(2\pi/k)$ and $\mathrm{P}(\underline{\mathrm{e}}_2)$ (mirror transformation through the x axis),
- $\mathrm{O}(2)$: orthogonal group.

	Biclinic	Orthotropic	Tetragonal	Isotropic
$[\mathrm{G}_{\mathbb{C}}]$	$[\mathrm{Z}_2]$	$[\mathrm{D}_2]$	$[\mathrm{D}_4]$	$[\mathrm{O}(2)]$
$\#_{\text{indep}}(\mathbb{C})$	6 (5)	4	3	2

The polar components

Following the work of Verchery, we introduce non polynomial quantities, the polar components, to represent the Cartesian components.

Example of 2nd order symmetric tensors

$$T = \frac{L_{11} + L_{22}}{2}, \qquad Re^{2i\Phi} = \frac{L_{11} - L_{22}}{2} + iL_{12}$$

$$L_{11} = T + R\cos 2\Phi$$
$$L_{22} = T - R\cos 2\Phi$$
$$L_{12} = R\sin 2\Phi$$

- T, R, and Φ are the so-called polar components
- T and R^2 are polynomial invariants
- Φ is an angle that determines the tensor orientation
- T represents the spherical part and $Re^{2i\Phi}$ the deviatoric part

Case of 4th order elasticity tensor [Verchery 1979]

$$
\begin{aligned}
T_0 &= \frac{1}{8}(\mathbb{E}_{1111} - 2\mathbb{E}_{1122} + 4\mathbb{E}_{1212} + \mathbb{E}_{2222}) \\
T_1 &= \frac{1}{8}(\mathbb{E}_{1111} + 2\mathbb{E}_{1122} + \mathbb{E}_{2222}) \\
R_0 e^{4i\Phi_0} &= \frac{1}{8}\left[\mathbb{E}_{1111} - 2\mathbb{E}_{1122} - 4\mathbb{E}_{1212} + \mathbb{E}_{2222} + 4i(\mathbb{E}_{1112} - \mathbb{E}_{12222})\right] \\
R_1 e^{2i\Phi_1} &= \frac{1}{8}\left[\mathbb{E}_{1111} - \mathbb{E}_{2222} + 2i(\mathbb{E}_{1112} + \mathbb{E}_{1222})\right]
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}_{1111} &= T_0 + 2T_1 + R_0\cos 4\Phi_0 + 4R_1\cos 2\Phi_1 \\
\mathbb{E}_{1112} &= R_0\sin 4\Phi_0 + 2R_1\sin 2\Phi_1 \\
\mathbb{E}_{1122} &= -T_0 + 2T_1 - R_0\cos 4\Phi_0 \\
\mathbb{E}_{1212} &= T_0 - R_0\cos 4\Phi_0 \\
\mathbb{E}_{1222} &= -R_0\sin 4\Phi_0 + 2R_1\sin 2\Phi_1 \\
\mathbb{E}_{2222} &= T_0 + 2T_1 + R_0\cos 4\Phi_0 - 4R_1\cos 2\Phi_1
\end{aligned}
$$

Transformation of a 4th order elasticity tensor

$$R(\theta) : \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad P(\underline{e}_2) : \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathbb{E} = (T_0, T_1, R_0 e^{4i\Phi_0}, R_1 e^{2i\Phi_1})$$

$$R(\theta) \star \mathbb{E} = (T_0, T_1, R_0 e^{4i(\Phi_0+\theta)}, R_1 e^{2i(\Phi_1+\theta)})$$

$$P(\underline{e}_1) \star \mathbb{E} = (T_0, T_1, R_0 e^{4i\Phi_0}, R_1 e^{-2i\Phi_1})$$

2D elasticity tensor invariants

O(2)-integrity basis of $\mathbb{E}\mathrm{la}$

The following quantities

$$I_1 = T_1 \quad J_1 = T_0 \quad I_2 = R_1^2 \quad J_2 = R_0^2 \quad I_3 = R_0 R_1^2 \cos 4(\Phi_0 - \Phi_1)$$

- Constitute an integrity basis for the O(2)-action;
- The algebra $\mathbb{R}[\mathbb{E}\mathrm{la}]^{\mathrm{O}(2)}$ is free.

2D Symmetry classes

- Isotropy

$$R_0 = 0 \text{ and } R_1 = 0$$

- Square symmetry (tetragonal)

$$R_1 = 0$$

- Orthotropy

$$\cos 4(\Phi_0 - \Phi_1) = \pm 1 \quad \Leftrightarrow \quad \Phi_0 - \Phi_1 = K\frac{\pi}{4} \quad K \in \{0, 1\}$$

- Anisotropy (biclinic)

Polar decomposition of the strain energy

- Putting the strain energy $V = \frac{1}{2}\boldsymbol{\sigma} : \boldsymbol{\varepsilon}$ in the form $V = V_{sph} + V_{dev}$

$$V_{sph} := \frac{1}{2}\boldsymbol{\varepsilon}_{sph} : \boldsymbol{\sigma}_{sph} = T\ t,$$
$$V_{dev} := \frac{1}{2}\boldsymbol{\varepsilon}_{dev} : \boldsymbol{\sigma}_{dev} = R\ r\cos 2(\Phi - \varphi),$$

  we get

$$V_{sph} = 4T_1 t^2 + 4R_1 r\ t\cos 2(\Phi_1 - \varphi),$$
$$V_{dev} = 2r^2\left[T_0 + R_0\cos 4(\Phi_0 - \varphi)\right] + 4R_1 r\ t\cos 2(\Phi_1 - \varphi).$$

- T_1 affects only V_{sph},
  T_0 and R_0 only V_{dev},
  while R_1 couples V_{sph} with V_{dev}.

- For materials with $R_1 = 0$, the two parts are uncoupled.

Bounds on the polar invariants

- The positive definiteness of $\mathbb{E}$ can be expressed in terms of bounds on its polar invariants
- It can be shown that the positive definiteness reduces to the following

$$T_0 - |R_0| > 0,$$
$$T_1(T_0^2 - R_0^2) - 2R_1^2\left[T_0 - R_0\cos 4(\Phi_0 - \Phi_1)\right] > 0$$

- The above conditions $\Rightarrow T_0 > 0, T_1 > 0$.

Polar parameters of the inverse tensor

- The polar components of $\mathbb{S}=\mathbb{E}^{-1}$, denoted by the lower-case letters t_0, t_1, r_0, r_1 and $\varphi_0-\varphi_1$, are:

$$
\begin{aligned}
t_0 &= \frac{2}{\Delta}\left(T_0T_1-R_1^2\right),\\
t_1 &= \frac{1}{2\Delta}\left(T_0^2-R_0^2\right),\\
r_0e^{4i\varphi_0} &= \frac{2}{\Delta}\left[\left(R_1e^{2i\Phi_1}\right)^2-T_1R_0e^{4i\Phi_0}\right],\\
r_1e^{2i\varphi_1} &= \frac{1}{\Delta}\left[R_0e^{4i\Phi_0}R_1e^{-2i\Phi_1}-T_0R_1e^{2i\Phi_1}\right].
\end{aligned}
$$

- Δ is an invariant positive quantity, defined by

$$\Delta=8T_1\left(T_0^2-R_0^2\right)-16R_1^2\left[T_0-R_0\cos 4\left(\Phi_0-\Phi_1\right)\right]$$

- An important result for the symmetry analysis is the fact that

$$R_1=0\Leftrightarrow r_1=0,\qquad R_0=0\not\Leftrightarrow r_0=0.$$

Are all the symmetries mechanically equivalent?

Let distinguish between ordinary orthotropies and a special orthotropy:

- $\Phi_0 - \Phi_1 = K\frac{\pi}{4}$ **and** $R_0 \neq 0$ $\rightarrow$ ordinary orthotropies ($K \in \{0, 1\}$) (determined by a polynomial cubic invariant);
- $R_0 = 0 \rightarrow$ special orthotropy (determined by a polynomial quadratic invariant).

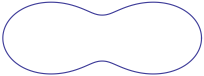
Ordinary orthotropies

- For the same set of invariants T_0, T_1, R_0 and R_1 two possible and distinct ordinary orthotropies exist: one with $K = 0$ and the other one with $K = 1$.
- For ordinary orthotropic materials

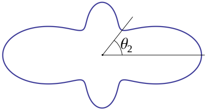
$$
\begin{aligned}
\mathbb{E}_{1111} &= T_0 + 2T_1 + (-1)^K R_0 \cos 4\Phi_1 + 4R_1 \cos 2\Phi_1 \\
\mathbb{E}_{1112} &= R_0 \sin 4\Phi_1 + 2R_1 \sin 2\Phi_1 \\
\mathbb{E}_{1122} &= -T_0 + 2T_1 - (-1)^K R_0 \cos 4\Phi_1 \\
\mathbb{E}_{1212} &= T_0 - (-1)^K R_0 \cos 4\Phi_1 \\
\mathbb{E}_{1222} &= -(-1)^K R_0 \sin 4\Phi_1 + 2R_1 \sin 2\Phi_1 \\
\mathbb{E}_{2222} &= T_0 + 2T_1 + (-1)^K R_0 \cos 4\Phi_1 - 4R_1 \cos 2\Phi_1
\end{aligned}
$$

- $\mathbb{E}_{1111}$ can be of 3 types, depending upon K and $\rho = \dfrac{R_0}{R_1}$

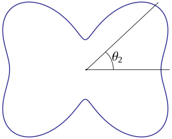
$\rho < 1, K = 0, 1$



$\rho > 1, K = 0$



$\rho > 1, K = 1$



- The type of ordinary orthotropy is not necessarily the same for stiffness and compliance:

$$\left.\begin{array}{l} K=0 \;\text{ and }\; R_1^2 > T_1 R_0 \\ \text{or} \\ K=1 \end{array}\right\} \;\Rightarrow\; k=0,$$

$$K=0 \;\text{ and }\; R_1^2 < T_1 R_0 \;\Rightarrow\; k=1.$$

the combination $(K=1, k=1)$ cannot exist.

- The value of K strongly affects the solution of an optimal design with in anisotropic elasticity: switching the value of K transforms the best in the worst solution (or viceversa).

R_0 SPECIAL-ORTHOTROPY

- $R_0 = 0$ identifies the so-called $R_0 - orthotropy$
- With $R_0 = 0$, we get

$$
\begin{aligned}
\mathbb{E}_{1111} &= T_0 + 2T_1 + 4R_1 \cos 2\Phi_1 \\
\mathbb{E}_{1112} &= + 2R_1 \sin 2\Phi_1 \\
\mathbb{E}_{1122} &= -T_0 + 2T_1 \\
\mathbb{E}_{1212} &= T_0 \\
\mathbb{E}_{1222} &= + 2R_1 \sin 2\Phi_1 \\
\mathbb{E}_{2222} &= T_0 + 2T_1 - 4R_1 \cos 2\Phi_1
\end{aligned}
$$

- Two components are isotropic,
  the others rotates like those of a 2nd order tensor.

- Because $R_0 = 0 \not\Leftrightarrow r_0 = 0$, the dual case exists too: r_0-orthotropy.
- It concerns the compliance tensor $\mathbb{S}$. In such a case, it can be shown that

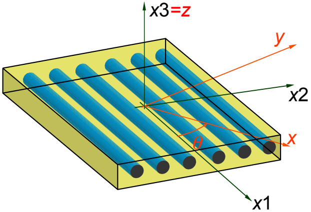
$$R_0 = \frac{R_1^2}{T_1}, \quad K = 0.$$

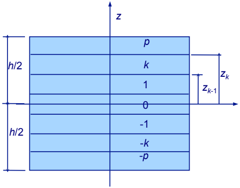
- It is interesting to notice that just the above relations distinguish this case from that of ordinary orthotropy of $\mathbb{E}$, otherwise indetectable.
- The invariance of S_{1212} implies that of G_{12}:

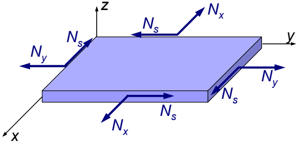
$$G_{12} = \frac{1}{4S_{1212}} = \frac{1}{4t_0}.$$

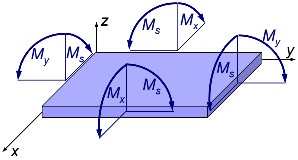
- This is the special orthotropy typical of paper (Vannucci, J Elas, 2010)

Abridged laminate mechanics









$$\left\{\frac{\mathbf{N}}{\mathbf{M}}\right\} = \left[\begin{array}{c|c} h\mathbb{A} & \frac{h^2}{2}\mathbb{B} \\ \hline \frac{h^2}{2}\mathbb{B} & \frac{h^3}{12}\mathbb{D} \end{array}\right] \left\{\frac{\boldsymbol{\varepsilon}}{\chi}\right\} \quad \Leftrightarrow \quad \left\{\frac{\boldsymbol{\varepsilon}}{\chi}\right\} = \left[\begin{array}{c|c} \frac{1}{h}\mathcal{A} & \frac{2}{h^2}\mathcal{B} \\ \hline \frac{2}{h^2}\mathcal{B}^\top & \frac{12}{h^3}\mathcal{D} \end{array}\right] \left\{\frac{\mathbf{N}}{\mathbf{M}}\right\}$$

The stiffness tensors

$$\mathbb{A} \rightarrow \begin{cases} T_0^A = T_0 \\ T_1^A = T_1 \\ R_0^A e^{4i\Phi_0^A} = R_0 e^{4i\Phi_0} (\xi_1 + i\xi_3) \\ R_1^A e^{2i\Phi_1^A} = R_1 e^{2i\Phi_1} (\xi_2 + i\xi_4) \end{cases}$$

$$\mathbb{B} \rightarrow \begin{cases} T_0^B = 0 \\ T_1^B = 0 \\ R_0^B e^{4i\Phi_0^B} = R_0 e^{4i\Phi_0} (\xi_5 + i\xi_7) \\ R_1^B e^{2i\Phi_1^B} = R_1 e^{2i\Phi_1} (\xi_6 + i\xi_8) \end{cases}$$

$$\mathbb{D} \rightarrow \begin{cases} T_0^D = T_0 \\ T_1^D = T_1 \\ R_0^D e^{4i\Phi_0^D} = R_0 e^{4i\Phi_0} (\xi_9 + i\xi_{11}) \\ R_1^D e^{2i\Phi_1^D} = \underbrace{R_1 e^{2i\Phi_1}}_{\text{material}} \underbrace{(\xi_{10} + i\xi_{12})}_{\text{geometry}} \end{cases}$$

Lamination parameters

$$\begin{cases} \xi_1 + i\xi_3 = \frac{1}{n} \sum_{j=1}^{n} e^{4i\delta_j} \\ \xi_2 + i\xi_4 = \frac{1}{n} \sum_{j=1}^{n} e^{2i\delta_j} \end{cases}$$

$$\begin{cases} \xi_5 + i\xi_7 = \frac{1}{n^2} \sum_{j=1}^{n} b_j \; e^{4i\delta_j} \\ \xi_6 + i\xi_8 = \frac{1}{n^2} \sum_{j=1}^{n} b_j \; e^{2i\delta_j} \end{cases}$$

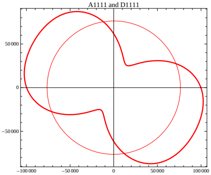
$$\begin{cases} \xi_9 + i\xi_{11} = \frac{1}{n^3} \sum_{j=1}^{n} d_j \; e^{4i\delta_j} \\ \xi_{10} + i\xi_{12} = \frac{1}{n^3} \sum_{j=1}^{n} d_j \; e^{2i\delta_j} \end{cases}$$

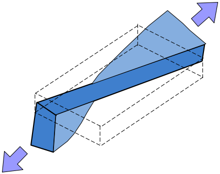
Effects of homogenization on anisotropy

- The definition of the anisotropy behavior and of elastic symmetries can be problematic in laminates:

- Generally speaking, $\mathbb{A} \neq \mathbb{D}$ and $\mathcal{A} \neq \mathcal{D}$

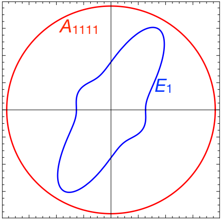
- $\mathbb{B} \neq 0$ →coupling





- While $\mathbb{B} = \mathbb{B}^\top$, this is not true for compliance: $\mathcal{B} \neq \mathcal{B}^\top$.

- If $\mathbb{B} \neq 0$, the symmetries of $\mathbb{A}$ and $\mathbb{D}$ are lost for $\mathcal{A}$ and $\mathcal{D}$



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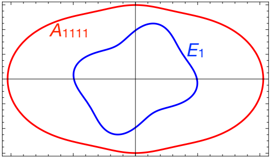

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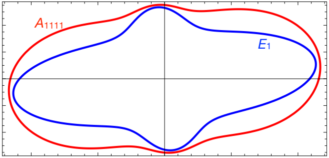
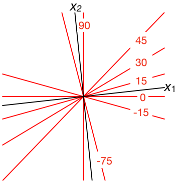
- An elastic symmetry can exist also without any material symmetry. An example with $\mathbb{A}$ orthotropic, $\mathbb{B} = 0$, $\mathbb{D}$ anisotropic: [0/30/-15/15/90/-75/0/45/-75/0/-15/15]

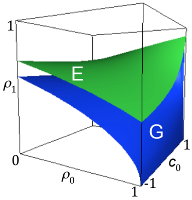




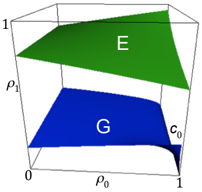
- Geometrical bounds for polar components of a laminate (Vannucci, J Elas, 2013)

$$\rho = \frac{R_0}{R_1}, \quad \rho_0 = \frac{R_0^{A,D}}{R_0}, \quad \rho_1 = \frac{R_1^{A,D}}{R_1}, \quad \tau_0 = \frac{T_0}{R_0}, \quad \tau_1 = \frac{T_1}{R_1}.$$

$$0 \le \rho_0, \ \ 0 \le \rho_1, \ \ \rho_0 \le 1, \ \ 2\rho_1^2 \le \frac{1-\rho_0^2}{1-(-1)^{K^L}\rho_0\ c_0}, \ \ 2\rho_1^2 < \rho\tau_0\tau_1 \frac{1-\left(\frac{\rho_0}{\tau_0}\right)^2}{1-\frac{\rho_0}{\tau_0}\ c_0}.$$



Carbon-epoxy T-300/5208



Braided carbon-epoxy BR45-a

Design of anisotropic laminates

- Design problem → optimization of a cost function: $\min_x f(x)$
  - x: design variables (typically: n, δ_j, thicknesses etc.)
  - usually, the material is chosen *a priori* → the isotropic part is determined by this choice
- Be: $\mathcal{P} = \{\mathcal{P}_i,\ i = 1, ..., 12\} = \{R_0, R_1, \Phi_0 - \Phi_1, \Phi_1\}_{A,B,D}$
  - $\mathbb{A} = \mathbb{A}(\mathcal{P}_i),\ \mathbb{B} = \mathbb{B}(\mathcal{P}_i),\ \mathbb{D} = \mathbb{D}(\mathcal{P}_i)$, unique correspondence
  - functions $\mathcal{P}_i = \mathcal{P}_i(\delta_j)$ are not bijective
- Each problem is split into 2 subproblems, linked together and to be solved in sequence:
  - Step 1: the Structure Problem: design of the optimal anisotropy properties with respect to $f(x)$; the problem is formulated in the space of the $\mathcal{P}_i$s;
  - Step 2: the Constitutive Law Problem: determination of a suitable stacking sequence δ_j able to realize a laminate with the optimal $\mathcal{P}_i$s; non-bijectivity ⇒ non-uniqueness.

The advantages of the polar method

- The use of the polar parameters automatically eliminates the redundant design variables, because

$$\mathbb{A} \rightarrow \begin{cases} T_0^A = T_0 \\ T_1^A = T_1 \\ R_0^A e^{4i\Phi_0^A} = R_0 e^{4i\Phi_0} (\xi_1 + i\xi_3) \\ R_1^A e^{2i\Phi_1^A} = R_1 e^{2i\Phi_1} (\xi_2 + i\xi_4) \end{cases} \quad \mathbb{D} \rightarrow \begin{cases} T_0^D = T_0 \\ T_1^D = T_1 \\ R_0^D e^{4i\Phi_0^D} = R_0 e^{4i\Phi_0} (\xi_9 + i\xi_{11}) \\ R_1^D e^{2i\Phi_1^D} = R_1 e^{2i\Phi_1} (\xi_{10} + i\xi_{12}) \end{cases}$$

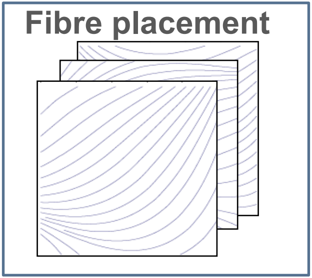
- The size of Step 1 problem is fixed and independent from the number of plies; e.g. for orthotropic laminates made of identical plies, it is of only 3 polar parameters ∀ tensor:

$$R_{0K} = (-1)^K R_0, R_1, \Phi_1$$

- The geometrical constraints are known in explicit form for the polar parameters, ∀ type of anisotropy

Optimal anisotropic fields

- Idea: fibre placement
- Pb: properties (p. ex. $\mathbb{B} = 0$) are local
- Mathematically: optimization of three tensor fields of anisotropy, with local constraints

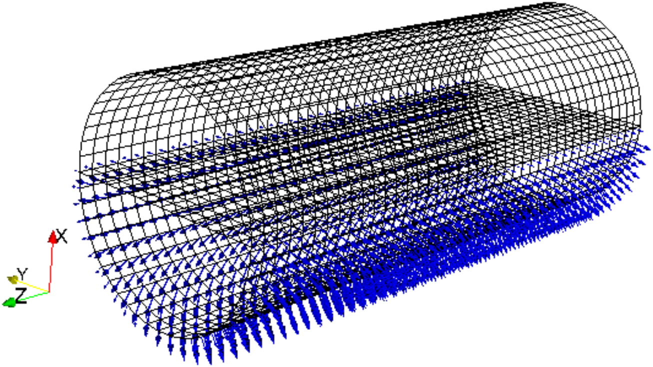


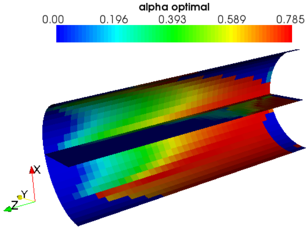
- Problems considered up till now:
  - stiffness optimization

    (PhD theses of C. Julien and A. Jibawy, 2010, Univ P6)
  - stiffness and strength optimization

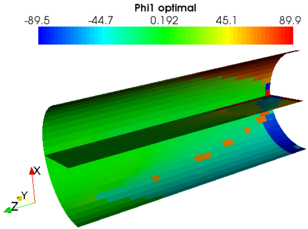
    (PhD thesis of A. Catapano, 2013, Univ P6)

- Example 1: optimization of an aircraft-like structure
- Objective: minimization of the compliance; angle-ply laminates

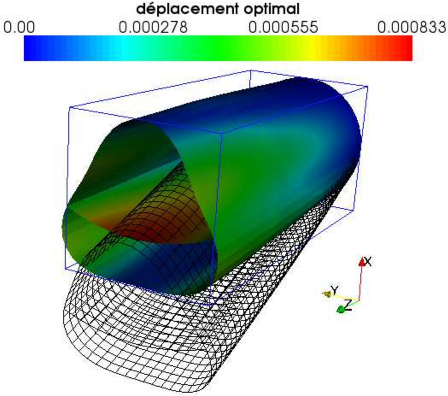




(a) Répartition de l'orientation optimale des fibres α^{opt}

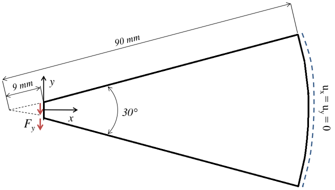


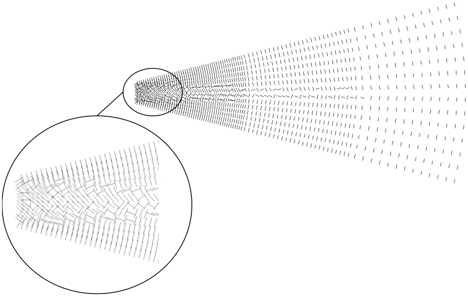
(b) Répartition des valeurs de Φ_1^{opt}

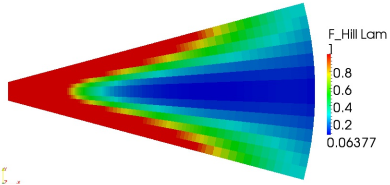


- Example 2: maximize stiffness and strength; for a laminate having the minimal compliance, determine the highest strength (or viceversa).

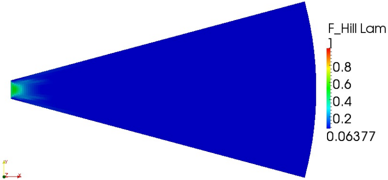
  A. Catapano: Stiffness and strength optimisation of the anisotropy distribution for laminated structures. PhD thesis, Univ P6, 2013







a)



b)

Designing topology and anisotropy

- Industrial research (Stelia and ONERA) (PhD thesis of N. Ranaivomiarana, 2019)
- Problem: determine the optimal topology and field of anisotropy for a 2D or 3D structure to be designed with respect to a given criterion and constraints
- Interaction between anisotropy and geometry.