

A deterministic approximation method in shape optimization under random uncertainties

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Foreword: uncertainties in structural optimization

- Mechanical systems rely on **data**, e.g. the loads, the properties of a constituent material, or the geometry of the system itself.
- In concrete situations, such data are plagued with **uncertainties** because:
 - they may be available only through (error-prone) measurements,
 - they may be altered with time (wear) and conditions of the ambient medium.
- The performances of structures are **very sensitive** to small perturbations of data.

⇒ **Need to somehow anticipate uncertainties when designing and optimizing shapes.**



A disk braking system



A worn out braking pad

1 Introduction and definitions

- Foreword
- The main ideas in an abstract framework

2 Applications in shape optimization

- Shape optimization of elastic structures
- Shape optimization under random loads
- Shape optimization under uncertainties on the elastic material
- Shape optimization under geometric uncertainties

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The main ideas in an abstract framework (I)

- $\mathcal{U}_{ad} \subset \mathcal{H}$ is a set of **admissible designs** h (e.g. the thickness of a plate, the geometry of a shape).
- $(\mathcal{P}, || \cdot ||)$ is a Banach space of **data** f (forces, parameters of a material).
- The performances of a design h are evaluated in terms of a **cost** $\mathcal{C} \equiv \mathcal{C}(f, u_{h,f})$, which involves a **state** $u_{h,f}$, solution to a **physical system**:

$$\mathcal{A}(h)u_{h,f} = b(f),$$

where f acts on the right-hand side for simplicity.

- The data are **uncertain**, and read:

$$f = f_0 + \hat{f}(\omega),$$

where f_0 is a mean value, and ω is an event, in an abstract probability space $(\mathcal{O}, \mathcal{F}, \mathbb{P})$.

The main ideas in an abstract framework (II)

There are two different settings to deal with uncertainties:

- Worst-case approach: When only a maximum bound $\|\hat{f}\|_{\mathcal{P}} \leq m$ is available on perturbations, one considers the **worst-case functional**:

$$\mathcal{J}_{wc}(h) = \sup_{\|\hat{f}\|_{\mathcal{P}} \leq m} \mathcal{C}(f_0 + \hat{f}, u_{h, f_0 + \hat{f}}).$$

Main drawback: **Pessimistic** approach, which may yield designs with unnecessarily bad nominal performances.

- Probabilistic approach: When information is available on the moments of the uncertainties, one may try to minimize the **mean value**:

$$\mathcal{M}(h) = \int_{\mathcal{O}} \mathcal{C}(f_0 + \hat{f}(\omega), u_{h, f_0 + \hat{f}(\omega)}) \mathbb{P}(d\omega),$$

or a **failure probability**:

$$\mathcal{P}(h) = \mathbb{P} \left(\left\{ \omega \in \mathcal{O}, \mathcal{C} \left(f_0 + \hat{f}(\omega), u_{h, f_0 + \hat{f}(\omega)} \right) > \alpha \right\} \right).$$

The main ideas in an abstract framework (III)

Working hypotheses:

- Perturbations are **small**: depending on the context, this may mean:
 - $\hat{f} \in L^\infty(\mathcal{O}, \mathcal{P})$: all the realizations $\hat{f}(\omega) \in \mathcal{P}$ are small.
 - $\hat{f} \in L^p(\mathcal{O}, \mathcal{P})$, for $p < \infty$: $\hat{f}$ may have **unprobably** large realizations.
- Perturbations are **finite-dimensional**:

$$\hat{f}(\omega) = \sum_{i=1}^N f_i \xi_i(\omega),$$

where $f_i \in \mathcal{P}$, and the ξ_i are normalized, uncorrelated random variables:

$$\int_{\mathcal{O}} \xi_i(\omega) \mathbb{P}(d\omega) = 0, \quad \int_{\mathcal{O}} \xi_i(\omega) \xi_j(\omega) \mathbb{P}(d\omega) = \delta_{i,j}.$$

Example: $\hat{f}$ is obtained as a truncated **Karhunen-Loève expansion**.

The main ideas in an abstract framework (IV)

Strategy:

- Calculate approximate functionals $\widetilde{\mathcal{M}}(h)$ and $\widetilde{\mathcal{P}}(h)$, which are
 - **deterministic**: no random variable or probabilistic integral is involved.
 - **consistent** with their exact counterparts, i.e. the differences $|\mathcal{M}(h) - \widetilde{\mathcal{M}}(h)|$ and $|\mathcal{P}(h) - \widetilde{\mathcal{P}}(h)|$ are 'small'.
- Calculate their derivatives $\widetilde{\mathcal{M}}'(h)(\widehat{h})$ and $\widetilde{\mathcal{P}}'(h)(\widehat{h})$,
- **Minimize** the approximate functionals $\widetilde{\mathcal{M}}(h)$ and $\widetilde{\mathcal{P}}(h)$ (under constraints), by using the expressions of their derivatives.
e.g. relying on a **steepest-descent algorithm**.

The main ideas in an abstract framework (V)

Use the **smallness** of perturbations to perform a first- or second-order Taylor expansion of the mappings $f \mapsto u_{h,f}$ and $f \mapsto \mathcal{C}(f, u_{h,f})$ around f_0 :

$$u_{h,f_0+\hat{f}} \approx u_h + u_h^1(\hat{f}) + \frac{1}{2}u_h^2(\hat{f}, \hat{f}),$$

where $\mathcal{A}(h)u_h^1(\hat{f}) = \frac{\partial b}{\partial f}(f_0)(\hat{f})$, and $\mathcal{A}(h)u_h^2(\hat{f}, \hat{f}) = \frac{\partial^2 b}{\partial f^2}(f_0)(\hat{f}, \hat{f})$.

$$\mathcal{C}(f_0 + \hat{f}, u_{h,f_0+\hat{f}}) \approx \mathcal{C}(f_0, u_h) + \mathcal{L}_h(\hat{f}) + \frac{1}{2}\mathcal{B}_h(\hat{f}, \hat{f}),$$

where the linear and bilinear forms $\mathcal{L}_h$ and $\mathcal{B}_h$ read:

$$\mathcal{L}_h(\hat{f}) = \frac{\partial \mathcal{C}}{\partial f}(f_0, u_h)(\hat{f}) + \frac{\partial \mathcal{C}}{\partial u}(f_0, u_h)(u_h^1(\hat{f})),$$

$$\begin{aligned} \mathcal{B}_h(\hat{f}, \hat{f}) = & \frac{\partial^2 \mathcal{C}}{\partial f^2}(f_0, u_h)(\hat{f}, \hat{f}) + 2 \frac{\partial^2 \mathcal{C}}{\partial f \partial u}(f_0, u_h)(\hat{f}, u_h^1(\hat{f})) \\ & + \frac{\partial^2 \mathcal{C}}{\partial u^2}(f_0, u_h)(u_h^1(\hat{f}), u_h^1(\hat{f})) + \frac{\partial \mathcal{C}}{\partial u}(f_0, u_h)(u_h^2(\hat{f}, \hat{f})). \end{aligned}$$

Approximation of moment functionals

- Replacing the cost with its second-order expansion gives rise to the **approximate mean-value** functional:

$$\widetilde{\mathcal{M}}(h) = \mathcal{C}(f_0, u_h) + \int_{\mathcal{O}} \mathcal{L}_h(\widehat{f}(\omega)) \mathbb{P}(d\omega) + \frac{1}{2} \int_{\mathcal{O}} \mathcal{B}_h(\widehat{f}(\omega), \widehat{f}(\omega)) \mathbb{P}(d\omega).$$

- Using the structure of perturbations $\widehat{f}(\omega) = \sum_{i=1}^N f_i \xi_i(\omega)$, it comes:

$$\widetilde{\mathcal{M}}(h) = \mathcal{C}(f_0, u_h) + \frac{1}{2} \sum_{i=1}^N \mathcal{B}_h(f_i, f_i),$$

a formula which involves the calculation of the $N + 2$ ‘reduced states’:

$$u_h, \quad u_{h,i} := u_h^1(f_i), \quad (i = 1, \dots, N), \quad \text{and} \quad u_h^2 := \sum_{i=1}^N u_h^2(f_i, f_i).$$

- This approach can be applied to other moments of $\mathcal{C}$, e.g. its **variance**:

$$\mathcal{V}(h) = \int_{\mathcal{O}} \left(\mathcal{C}(f_0 + \widehat{f}(\omega), u_{h, f_0 + \widehat{f}(\omega)}) - \mathcal{M}(h) \right)^2 \mathbb{P}(d\omega).$$

Approximation of failure probabilities (I)

Additional hypotheses: The random variables ξ_i are:

- **independent**,
- **Gaussian**, i.e. their cumulative distribution function is:

$$\mathbb{P}(\{\omega \in \mathcal{O}, \xi_i(\omega) < \alpha\}) = \Phi(\alpha) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha} e^{-\frac{\xi^2}{2}} d\xi.$$

The (exact) failure probability reads:

$$\mathcal{P}(h) = \frac{1}{(2\pi)^{N/2}} \int_{\mathcal{D}(h)} e^{-\frac{|\xi|^2}{2}} d\xi,$$

where the **failure region** $\mathcal{D}(h)$ is:

$$\mathcal{D}(h) = \left\{ \xi \in \mathbb{R}^N, \mathcal{C} \left(f_0 + \sum_{i=1}^N f_i \xi_i, u_{h, f_0 + \sum_{i=1}^N f_i \xi_i} \right) > \alpha \right\}.$$

Approximation of failure probabilities (II)

Idea: Approximate the failure region with:

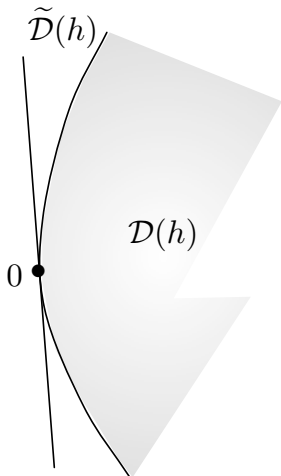
$$\tilde{\mathcal{D}}(h) = \left\{ \xi \in \mathbb{R}^N, \mathcal{C}(f_0, u_h) + \sum_{i=1}^N \mathcal{L}_h(f_i) \xi_i > \alpha \right\}.$$

The **approximate failure probability**

$$\tilde{\mathcal{P}}(h) = \frac{1}{(2\pi)^{N/2}} \int_{\tilde{\mathcal{D}}(h)} e^{-\frac{|\xi|^2}{2}} d\xi$$

can be calculated in closed form as:

$$\tilde{\mathcal{P}}(h) = \Phi \left(-\frac{\alpha - \mathcal{C}(f_0, u_h)}{\sqrt{\sum_{i=1}^N \mathcal{L}_h(f_i)^2}} \right).$$



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The usual linear elasticity setting

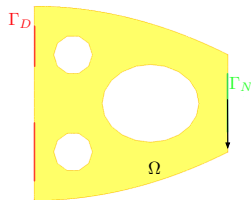
A **shape** is a bounded domain $\Omega \subset \mathbb{R}^d$, which is

- **fixed** on a part Γ_D of its boundary,
- submitted to **surface loads** g , applied on $\Gamma_N \subset \partial\Omega$, $\Gamma_D \cap \Gamma_N = \emptyset$.

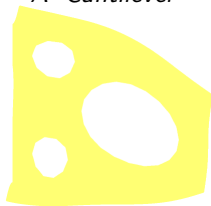
The displacement vector field $u_\Omega \in H_{\Gamma_D}^1(\Omega)^d$ is governed by the **linear elasticity system**:

$$\left\{ \begin{array}{ll} -\operatorname{div}(Ae(u_\Omega)) & = f \quad \text{in } \Omega \\ u_\Omega & = 0 \quad \text{on } \Gamma_D \\ \begin{array}{ll} Ae(u_\Omega)n & = g \quad \text{on } \Gamma_N \\ Ae(u_\Omega)n & = 0 \quad \text{on } \Gamma := \partial\Omega \setminus (\Gamma_D \cup \Gamma_N) \end{array} \end{array} \right. ,$$

where $e(u) = \frac{1}{2}(\nabla u^T + \nabla u)$ is the strain tensor, and A is the Hooke's law of the material.



A 'Cantilever'

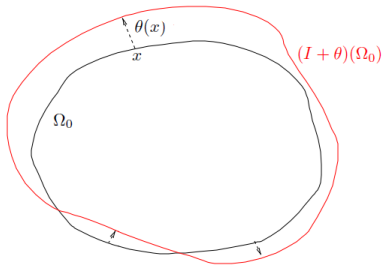


The deformed cantilever

Hadamard's boundary variation method describes variations of a reference, Lipschitz domain Ω of the form:

$$\Omega \rightarrow \Omega_\theta := (I + \theta)(\Omega),$$

for 'small' $\theta \in W^{1,\infty}(\mathbb{R}^d, \mathbb{R}^d)$.



In practice:

- We restrict to a set of **admissible shapes**:

$$\mathcal{U}_{ad} := \{ \Omega \subset \mathbb{R}^d \text{ is open, bounded and Lipschitz, } \Gamma_D \cup \Gamma_N \subset \partial\Omega \}.$$

- Deformations θ are assumed within the admissible set:

$$\Theta_{ad} := \{ \theta \in W^{1,\infty}(\mathbb{R}^d, \mathbb{R}^d), \text{ such that } \theta = 0 \text{ on } \Gamma_D \cup \Gamma_N \}.$$

Definition 1.

Given a smooth domain Ω , a functional $J(\Omega)$ of the domain is **shape differentiable** at Ω if the function

$$W^{1,\infty}(\mathbb{R}^d, \mathbb{R}^d) \ni \theta \mapsto J(\Omega_\theta)$$

is Fréchet-differentiable at 0, i.e. the following expansion holds around 0:

$$J(\Omega_\theta) = J(\Omega) + J'(\Omega)(\theta) + o(\|\theta\|_{W^{1,\infty}(\mathbb{R}^d, \mathbb{R}^d)}).$$

Shape derivatives can be computed using techniques from optimal control; in the case of 'many' functions of the domain $J(\Omega)$, they enjoy the **structure**:

$$J'(\Omega)(\theta) = \int_{\Gamma} v_{\Omega} \theta \cdot n \, ds,$$

where v_{Ω} is a scalar field depending on u_{Ω} , and possibly on an **adjoint state** p_{Ω} .

The generic numerical algorithm

This shape gradient provides a natural **descent direction** for functional J : *for instance*, defining θ as

$$\theta = -v_{\Omega} n$$

yields, for $t > 0$ sufficiently small (*to be found numerically*):

$$J((I + t\theta)(\Omega)) = J(\Omega) - t \int_{\Gamma} v_{\Omega}^2 ds + o(t) < J(\Omega)$$

Gradient algorithm: For $n = 0, \dots$ convergence,

1. Compute the solution u_{Ω^n} (and p_{Ω^n}) of the elasticity system on Ω^n .
2. Compute the shape gradient $J'(\Omega^n)$ thanks to the previous formula, and infer a descent direction θ^n for the cost functional.
3. **Advect** the shape Ω^n according to θ^n , so as to get $\Omega^{n+1} := (I + \theta^n)(\Omega^n)$.

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Shape optimization under random loads (I)

- We consider uncertainties on the **body forces** f ($\mathcal{P} = L^2(\mathbb{R}^d)^d$):

$$f(x) = f_0(x) + \widehat{f}(x, \omega), \text{ where } \widehat{f}(x, \omega) = \sum_{i=1}^N f_i(x) \xi_i(\omega) \in L^2(\mathcal{O}, L^2(\mathbb{R}^d)^d).$$

- The **cost function** is of the form:

$$\mathcal{C}(f, \Omega) = \int_{\Omega} j(f, u_{\Omega, f}) \, dx,$$

where $j : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{R}$ is smooth, satisfies growth conditions, and $u_{\Omega, f} \in H_{\Gamma_D}^1(\Omega)^d$ is the solution u of:

$$\left\{ \begin{array}{lll} -\operatorname{div}(Ae(u)) & = & f \quad \text{in } \Omega \\ u & = & 0 \quad \text{on } \Gamma_D \\ Ae(u)n & = & 0 \quad \text{on } \Gamma_N \\ Ae(u)n & = & 0 \quad \text{on } \Gamma \end{array} \right. .$$

Shape optimization under random loads (II)

The approximate mean value functional reads:

$$\begin{aligned}\widetilde{\mathcal{M}}(\Omega) &= \int_{\Omega} j(f_0, u_{\Omega}) \, dx + \frac{1}{2} \sum_{i=1}^N \int_{\Omega} \nabla_f^2 j(f_0, u_{\Omega})(f_i, f_i) \, dx \\ &+ \sum_{i=1}^N \int_{\Omega} \nabla_f \nabla_u j(f_0, u_{\Omega})(f_i, u_{\Omega,i}^1) \, dx + \frac{1}{2} \sum_{i=1}^N \int_{\Omega} \nabla_u^2 j(f_0, u_{\Omega})(u_{\Omega,i}^1, u_{\Omega,i}^1) \, dx,\end{aligned}$$

the $u_{\Omega,i}^1$ being the solutions of:
$$\left\{ \begin{array}{ll} -\operatorname{div}(Ae(u)) = f_i & \text{in } \Omega \\ u = 0 & \text{on } \Gamma_D \\ Ae(u)n = 0 & \text{on } \Gamma_N \\ Ae(u)n = 0 & \text{on } \Gamma \end{array} \right. .$$

Proposition 1.

Under the additional assumption that $\widehat{f} \in L^3(\mathcal{O}, L^3(\mathbb{R}^d)^d)$, there exists a constant $C > 0$ (depending on Ω) such that:

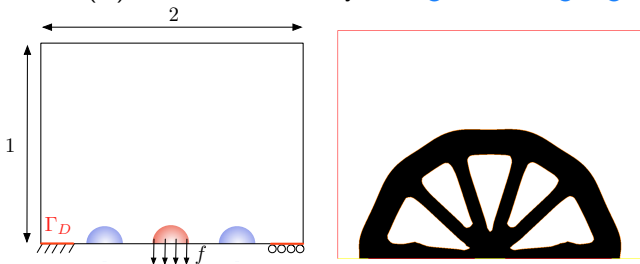
$$|\widetilde{\mathcal{M}}(\Omega) - \mathcal{M}(\Omega)| \leq C \|\widehat{f}\|_{L^3(\mathcal{O}, L^3(\mathbb{R}^d)^d)}^3.$$

Optimization of a bridge under random loads (I)

- The cost function is the **compliance** of shapes:

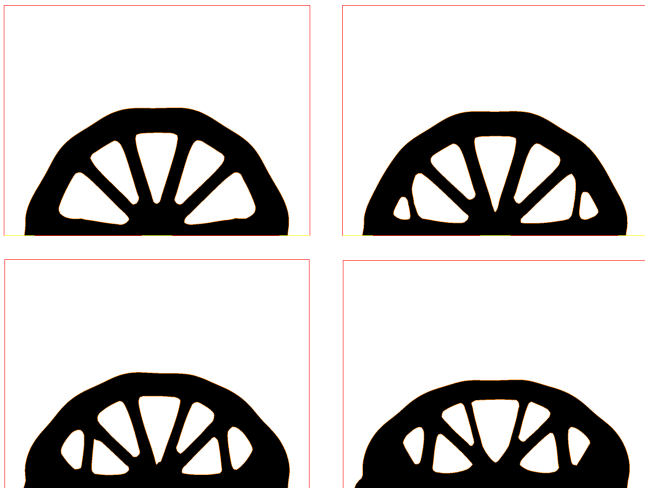
$$\mathcal{C}(f, \Omega) = \int_{\Omega} f \cdot u_{\Omega, f} \, dx = \int_{\Omega} Ae(u_{\Omega, f}) : e(u_{\Omega, f}) \, dx.$$

- Two load scenarios $f_1, f_2 = (0, -m)$ are supported in the blue spots.
- The considered objective function is: $\mathcal{L}(\Omega) = \widetilde{\mathcal{M}}(\Omega) + \delta \sqrt{\widetilde{\mathcal{V}}(\Omega)}$.
- A constraint $\text{Vol}(\Omega) = V_T$ is enforced by an **augmented Lagrangian algorithm**.



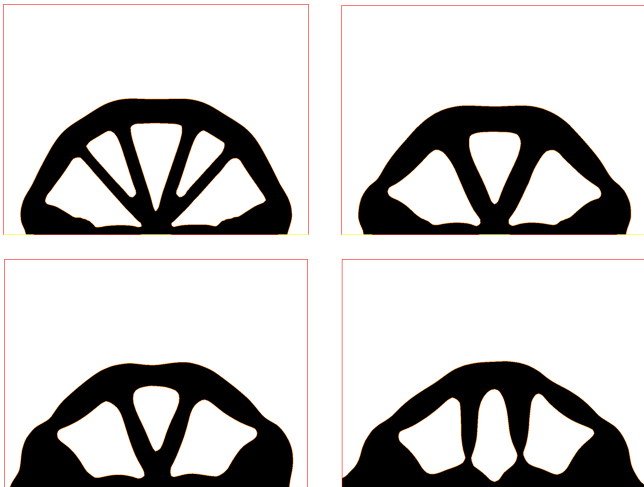
(Left) The bridge test case, (right) optimal shape in the unperturbed situation.

Optimization of a bridge under random loads (II)



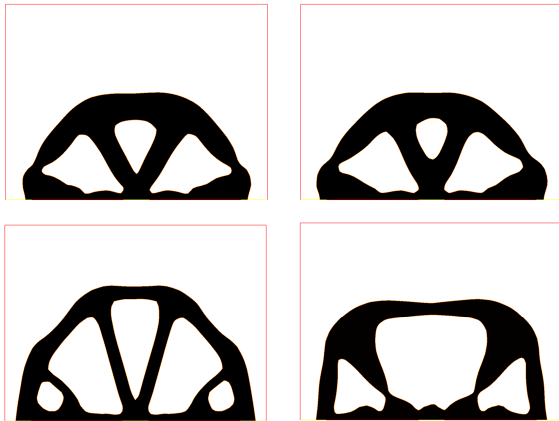
Optimal shapes for $\delta = 0$ and $m = 1, 2, 5, 10$.

Optimization of a bridge under random loads (III)



Optimal shapes for $\delta = 3$ and $m = 1, 2, 5, 10$.

Comparison with the worst-case approach



Optimal shapes for the linearized worst-case design approach with $m = 1, 2, 5, 10$.

Observation: The optimal shapes for the probabilistic functionals show systematically better nominal performances than their worst-case counterparts.

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Optimization under material uncertainties

- Perturbations over the **Young's modulus** E of the material are considered:

$$E = E_0 + \widehat{E}(x, \omega), \text{ where } \widehat{E}(x, \omega) = \sum_{i=1}^N E_i(x) \xi_i(\omega) \in L^\infty(\mathcal{O}, L^\infty(\mathbb{R}^d)).$$

- The cost function is of the form $\mathcal{C}(\Omega, \mathbf{E}) = \int_{\Omega} j(u_{\Omega}, \mathbf{E}) \, dx$, where:

$$\begin{cases} -\operatorname{div}(A(\mathbf{E})e(u_{\Omega})) &= 0 & \text{in } \Omega \\ u_{\Omega} &= 0 & \text{on } \Gamma_D \\ A(\mathbf{E})e(u_{\Omega})n &= g & \text{on } \Gamma_N \\ A(\mathbf{E})e(u_{\Omega})n &= 0 & \text{on } \Gamma \end{cases}.$$

- Minimization of the approximate **mean value** of $\mathcal{C}$:

$$\widetilde{\mathcal{M}}(\Omega) = \int_{\Omega} j(u_{\Omega}) \, dx + \frac{1}{2} \sum_{i=1}^N \int_{\Omega} \nabla^2 j(u_{\Omega})(u_{\Omega,i}^1, u_{\Omega,i}^1) \, dx + \frac{1}{2} \int_{\Omega} \nabla j(u_{\Omega}) \cdot u_{\Omega}^2 \, dx,$$

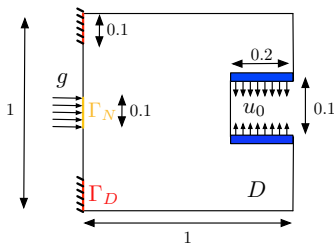
where the $u_{\Omega,i}^1$, $i = 1, \dots, N$, and u_{Ω}^2 are the **reduced states**.

Optimization of a grip under material uncertainties (I)

The cost function is

$$\mathcal{C}(\Omega, E) = \int_{\Omega} k(x) |u_{\Omega, E} - u_0|^2 dx,$$

where k is a localization factor, and u_0 is a target displacement, cooked so that the jaws close.



Setting of the gripping mechanism example.

Optimization of a grip under material uncertainties (II)

- The perturbations $\hat{E}(x, \omega)$ are known via their **two-point correlation** function:

$$\text{Cor}(\hat{E})(x, y) := \int_{\mathcal{O}} \hat{E}(x, \omega) \hat{E}(y, \omega) \mathbb{P}(d\omega) = \beta^2 e^{-\frac{|x-y|}{\ell}},$$

where β is a scaling factor, and ℓ is a characteristic length.

- A **Karhunen-Loève** expansion of $\hat{E}$ is performed, then truncated:

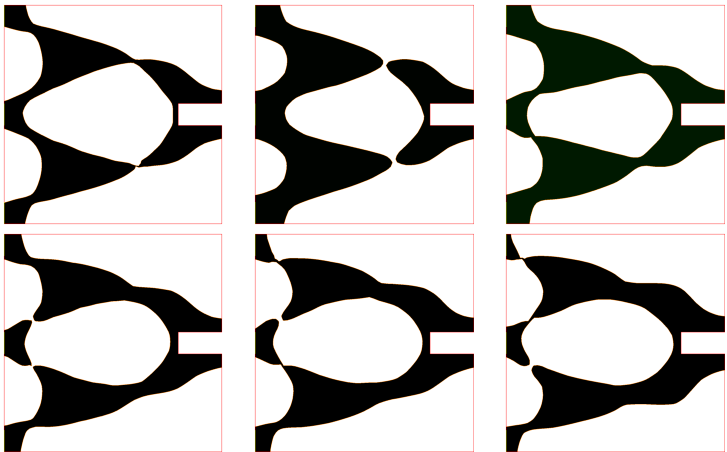
$$\hat{E}(x, \omega) \approx \sum_{i=1}^N \sqrt{\lambda_i} E_i(x) \xi_i(\omega),$$

where the (λ_i, E_i) are the eigenpairs of the Hilbert-schmidt operator

$$L^2(D) \ni f \mapsto \int_D \text{Cor}(\hat{E})(x, y) f(y) dx \in L^2(D),$$

and the $\xi_i(\omega) = \int_D \hat{E}(x, \omega) f_i(x) dx$ are normalized and uncorrelated random variables.

Optimization of a grip under material uncertainties (III)



Optimal shapes associated to values of $\beta = 0, 0.5, 1, 1.5, 2, 2.5$.

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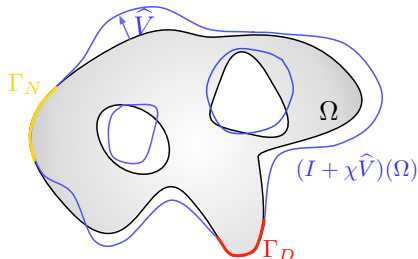
Modelling geometric uncertainties

Perturbations of a shape $\Omega \in \mathcal{U}_{ad}$ are considered with the structure:

$$\Omega \mapsto (I + \chi(x)\hat{v}(x, \omega)n_\Omega(x))(\Omega),$$

where:

- χ is a cutoff function, vanishing on $\Gamma_D \cup \Gamma_N$,
- n_Ω is (an extension of) the normal vector to $\partial\Omega$,
- The scalar field $\hat{v} \in L^\infty(\mathcal{O}, \mathcal{C}^{2,\infty}(\mathbb{R}^d))$ arises as $\hat{v}(x, \omega) = \sum_{i=1}^N v_i(x)\xi_i(\omega)$.



Perturbation of Γ by a vector field $\hat{V}$.

Optimization of a L-beam under geometric uncertainties

- The cost function is of the form:

$$\mathcal{C}(\Omega) = \int_{\Omega} j(\sigma(u_{\Omega})) \, dx,$$

where $\sigma(u) = Ae(u)$ is the **stress tensor**.

- The approximate **variance** functional reads:

$$\tilde{\mathcal{V}}(\Omega) = \sum_{i=1}^N a_{\Omega,i}^2 \text{ with } a_{\Omega,i} = \int_{\Gamma} (j(\sigma(u_{\Omega})) + Ae(u_{\Omega}) : e(p_{\Omega}) - f \cdot p_{\Omega}) \nu_i \, ds, \quad (1)$$

and the **adjoint state** $p_{\Omega} \in H_{\Gamma_D}^1(\Omega)^d$ is the solution of:

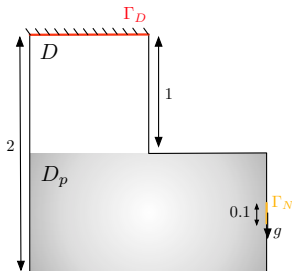
$$\begin{cases} -\operatorname{div}(Ae(p)) = \operatorname{div}\left(A \frac{\partial j}{\partial \sigma}(\sigma(u_{\Omega}))\right) & \text{in } \Omega, \\ p = 0 & \text{on } \Gamma_D, \\ Ae(p)n = -A \frac{\partial j}{\partial \sigma}(\sigma(u_{\Omega}))n & \text{on } \Gamma \cup \Gamma_N. \end{cases}$$

Optimization of a L-beam under geometric uncertainties

- Perturbations occur on a subregion $D_p \subset D$; their correlation function is:

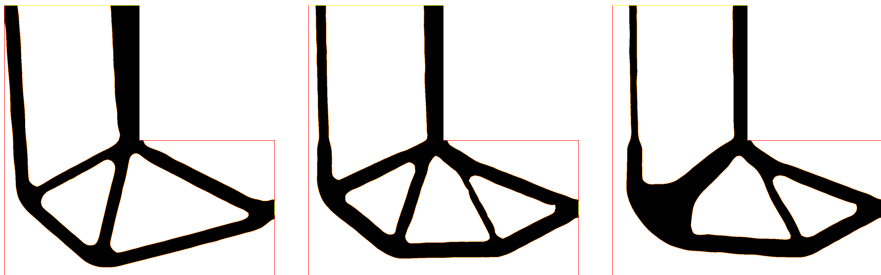
$$\text{Cor}(\hat{v})(x, \omega) = \beta^2 e^{-\frac{|x-y|}{\ell}}.$$

- The cost function is $\mathcal{C}(\Omega) = \int_{\Omega} \|\sigma(u_{\Omega})\|^5 dx$, and the objective $\mathcal{C}(\Omega) + \delta \sqrt{\tilde{v}(\Omega)}$ is minimized under a volume constraint.



Details of the L-shaped beam test-case.

Optimization of a L-beam under geometric uncertainties



Optimal shapes in the minimization of the stress-based criterion, where the parameter δ equals (from the left to the right) 0, 0.5, 2.

Thank you !

Thank you for your attention!

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