Advanced Calculus I: Workshop 12

Exercise 1

Let $a < b$ be two real numbers, and $c \in (a, b)$ be another point. Let $f : (a, b) \to \mathbb{R}$ be a function which is differentiable on (a, c) and (c, b), and such that:

$$\forall x \in (a, b) \setminus \{c\}, \ \ f'(x) > 0.$$

(1) Show that f does not admit any local extremum on (a, b).
(2) *Application:*Show that the function $f : \mathbb{R} \to \mathbb{R}$, defined by $x \mapsto x^3$ does not have any local extremum on $\mathbb{R}$.

Exercise 2

Let $f : \mathbb{R} \to \mathbb{R}$ be a function which is differentiable at 0, and such that:

$$\forall x, y \in \mathbb{R}, \ \ f(x + y) = f(x)f(y).$$

(1) Prove that f is differentiable on $\mathbb{R}$, and that its derivative satisfies:

$$\forall x \in \mathbb{R}, \ \ f'(x) = f'(0)f(x).$$

(2) Assuming the properties of the exponential function, infer that the function f is actually:

$$\forall x \in \mathbb{R}, \ \ f(x) = e^{cx},$$

where $c = f'(0)$.